

$$\begin{aligned}
\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) &= \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) \\
\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) &= (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c} \\
(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) &= (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c}) \\
\nabla \times \nabla \psi &= 0 \\
\nabla \cdot (\nabla \times \mathbf{a}) &= 0 \\
\nabla \times (\nabla \times \mathbf{a}) &= \nabla(\nabla \cdot \mathbf{a}) - \nabla^2 \mathbf{a} \\
\nabla \cdot (\psi \mathbf{a}) &= \mathbf{a} \cdot \nabla \psi + \psi \nabla \cdot \mathbf{a} \\
\nabla \times (\psi \mathbf{a}) &= \nabla \psi \times \mathbf{a} + \psi \nabla \times \mathbf{a} \\
\nabla(\mathbf{a} \cdot \mathbf{b}) &= (\mathbf{a} \cdot \nabla)\mathbf{b} + (\mathbf{b} \cdot \nabla)\mathbf{a} + \mathbf{a} \times (\nabla \times \mathbf{b}) + \mathbf{b} \times (\nabla \times \mathbf{a}) \\
\nabla \cdot (\mathbf{a} \times \mathbf{b}) &= \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b}) \\
\nabla \times (\mathbf{a} \times \mathbf{b}) &= \mathbf{a}(\nabla \cdot \mathbf{b}) - \mathbf{b}(\nabla \cdot \mathbf{a}) + (\mathbf{b} \cdot \nabla)\mathbf{a} - (\mathbf{a} \cdot \nabla)\mathbf{b}
\end{aligned} \tag{1}$$

If  $\mathbf{x}$  is the coordinate of a point with respect to some origin, with magnitude  $r = |\mathbf{x}|$ ,  $\mathbf{n} = \mathbf{x}/r$  is a unit radial vector, and  $f(r)$  is a well-behaved function of  $r$ , then

$$\begin{aligned}
\nabla \cdot \mathbf{x} &= 3 \quad \nabla \times \mathbf{x} = 0 \\
\nabla \cdot [\mathbf{n}f(r)] &= \frac{2}{r}f + \frac{\partial f}{\partial r} \quad \nabla \times [\mathbf{n}f(r)] = 0 \\
(\mathbf{a} \cdot \nabla)\mathbf{n}f(r) &= \frac{f(r)}{r}[\mathbf{a} - \mathbf{n}(\mathbf{a} \cdot \mathbf{n})] + \mathbf{n}(\mathbf{a} \cdot \mathbf{n})\frac{\partial f}{\partial r} \\
\nabla(\mathbf{x} \cdot \mathbf{a}) &= \mathbf{a} + \mathbf{x}(\nabla \cdot \mathbf{a}) + i(\mathbf{L} \times \mathbf{a})
\end{aligned} \tag{2}$$

where  $\mathbf{L} = \frac{1}{i}(\mathbf{x} \times \nabla)$  is the angular-momentum operator.