$\nabla\cdot(\vec{a}\cdot\vec{b})$

$\vec{a}=(a_1, a_2, a_3)$

$\vec{b}=(b_1, b_2, b_3)$

$\vec{a}\cdot\vec{b}=$ ~~a_ib_i~~

$a_ib_i = \delta_{ij}a_ib_j$

$\vec{a}\vec{b} = a_ib_j$

$\nabla\vec{a}$

$\nabla\cdot\vec{a} = \delta^{ij}\partial_i a_j$

$\nabla a = \partial_i a$

$\nabla\vec{a} = \partial_i a_j$

$\nabla(\vec{a}\vec{a}) = \nabla\cdot(a_ia_j)$

$= \partial_i a_i a_j$

$\nabla\cdot(\vec{a}\vec{b})$

$\partial_i a_i b_j$

$= b_j\partial_i a_i + a_i\partial_i b_j$

$= \vec{b}(\nabla\cdot\vec{a}) + (\vec{a}\cdot\nabla)\vec{b}$

$\nabla\cdot(\vec{a}\vec{b})$

$= \delta^{ij}\partial_i a_j b_k$

$= \delta^{ij}(a_j\partial_i b_k + b_k\partial_i a_j)$

$\nabla\cdot(a\vec{b}) \quad (\psi\vec{a})$

$\delta^{ij}a b_i$

$a\delta^i b_i + b_i\delta^i a$

$= a(\nabla\cdot\vec{b}) + (\vec{b}\cdot\nabla)a$

$= \psi(\nabla\cdot\vec{a}) + (\vec{a}\cdot\nabla)\psi$

$(\vec{a}\cdot\nabla)\vec{a} + \vec{a}(\nabla\cdot\vec{a})$

$= a_i\partial_i a_j + a_j\partial_i a_i$

$\nabla(\vec{a}\cdot\vec{b})$

$\partial_j(a_ib_i)$

$a_i\partial_j b_i + b_i\partial_j a_i$

$\vec{a}\times\vec{b} \quad \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \vec{i}(a_2b_3 - a_3b_2) + \vec{j}(a_3b_1 - a_1b_3) + \vec{k}(a_1b_2 - a_2b_1)$

$\varepsilon^{ijk}a_ib_j$

ε^{ijk}

$\varepsilon^{231}a_2b_3 + \varepsilon^{321}a_3b_2$

$1 \qquad -1$

$\varepsilon^{1jk}a_1 + \varepsilon^{2jk}a_2 + \varepsilon^{3jk}a_3$

$\vec{a}\times(\vec{b}\times\vec{c})$

$\varepsilon_{lkm}a^l\varepsilon^{ijk}b_ic_j$

$\varepsilon_{lkm}\varepsilon^{ijk}a^lb_ic_j$

$\vec{a}\cdot(\vec{b}\times\vec{c})$

$\delta_k{}^l a_l\varepsilon^{ijk}b_ic_j$

$\vec{b}\cdot(\vec{c}\times\vec{a})$

$\delta_k{}^l b_l\varepsilon^{ijk}c_ia_j$

$\delta_k{}^l\varepsilon^{ijk} \quad a_lb_ic_j \qquad a_jb_lc_i$

	j 1	2	3
k 1	0	$-a_3$	a_2
2	a_3	0	$-a_1$
3	$-a_2$	a_1	0

$\boxed{\varepsilon^{ijk}a_i}$

$\varepsilon_{lkm}\varepsilon^{ijk}$

$\delta^{ij}a_ic_jb_k - \delta^{ij}a_ib_jc_k$

~~$a_i(\delta^{ij}c_jb_k - \delta^{ij}c_kb_j)$~~

$\delta^{ij}a_i(c_jb_k - c_kb_j)$

$a_i = \delta^{ij}a_j \quad (\delta -1)$

$\varepsilon^{ijk}a_ib_j = (a_ib_j - a_jb_i)$

$\Omega \quad (a_2b_3 - a_3b_2)$

$\varepsilon_{kij}\varepsilon_{klm} = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})$

$$\varepsilon_{ijk}\varepsilon_{lmn} = \begin{array}{c|ccc|} l & \delta_{il} & \delta_{jl} & \delta_{kl} \\ m & \delta_{im} & \delta_{jm} & \delta_{km} \\ n & \delta_{in} & \delta_{jn} & \delta_{kn} \end{array}$$

$$\varepsilon_{ljk}\varepsilon_{lmn} = \begin{array}{c|ccc|} & l & j & k \\ l & \delta_{ll} & \delta_{jl} & \delta_{kl} \\ m & \delta_{lm} & \delta_{jm} & \delta_{km} \\ n & \delta_{ln} & \delta_{jn} & \delta_{kn} \end{array}$$

$$\varepsilon_{lmk}\varepsilon_{lmn} = \begin{array}{c|ccc|} & l & m & k \\ l & \delta_{ll} & \delta_{ml} & \delta_{kl} \\ m & \delta_{lm} & \delta_{mm} & \delta_{km} \\ n & \delta_{ln} & \delta_{mn} & \delta_{kn} \end{array}$$

$\varepsilon_{lmn}\varepsilon_{lmn} =$

$\delta_{ii'}\delta_{jj'}\delta_{kk} + \delta_{ji'}\delta_{kj'}\delta_{ik} + \delta_{ij'}\delta_{jk}\delta_{ki'}$

$-(\delta_{ki'}\delta_{jj'}\delta_{ik} + \delta_{kj'}\delta_{jk}\delta_{ii'} + \delta_{ji'}\delta_{ij'}\delta_{kk})$

$3\,\delta_{kk}(\delta_{ii'}\delta_{jj'} - \delta_{ji'}\delta_{ij'})$

$+\ \delta_{ik}(\delta_{ji'}\delta_{kj'} - \delta_{ki'}\delta_{jj'})$ 后面,跟上面一样,

$+\ \delta_{jk}(\delta_{ij'}\delta_{ki'} - \delta_{kj'}\delta_{ii'})$

$3-2=1$

若 $i=1,2,3,4$

$4-2=2$

$\varepsilon_{ijkl}\varepsilon_{i'j'k'l'} = \begin{array}{c|cccc} & i & j & k & l \\ i' & & & & \\ j' & & & & \\ k' & & & & \\ l' & & & & \end{array}$

$\varepsilon_{ijkl}\varepsilon_{ii'j'k'l'} = \begin{array}{c|cccc} & i & j & k & l \\ i' & & & & \\ j' & & & & \\ k' & & & & \\ l' & & & & \end{array}$ δ_{ll}

$$\varepsilon_{ij}\varepsilon_{i'j'} = \begin{array}{c|cc} & i & j \\ i' & \delta_{ii'} & \delta_{ji'} \\ j' & \delta_{ij'} & \delta_{jj'} \end{array}$$

指标数不同导致的.

$$\varepsilon_{ij}\varepsilon_{ij'} = \begin{array}{c|cc} & i & j \\ i & \delta_{ii} & \delta_{ji} \\ j' & \delta_{ij'} & \delta_{jj'} \end{array} = \delta_{jj'}$$

$\frac{d\vec{l}}{dl} = \vec{e} = \frac{\vec{B}}{B}\vec{e}$

$\frac{d\vec{l}}{B} = \frac{dl}{B}\vec{e}$

$$\varepsilon_{ij}\varepsilon_{ij} = \begin{array}{cc} \delta_{ii} & \delta_{ji} \\ \delta_{ij} & \delta_{jj} \end{array}$$